IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M06/5/MATME/SP2/ENG/TZ1/XX+

22067302

MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 4 May 2006 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 21]*

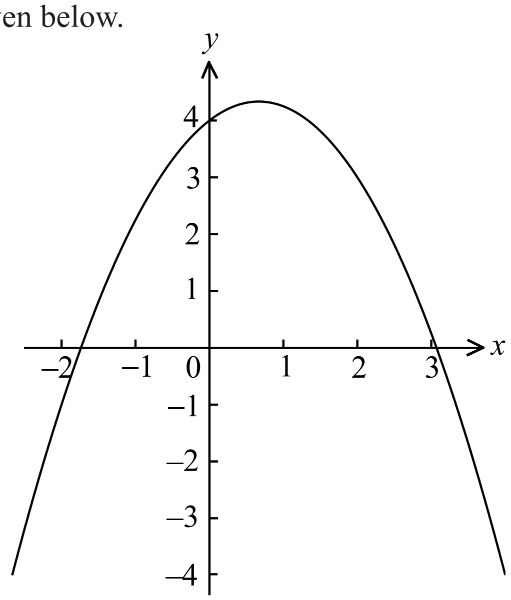
Let $f(x) = -\frac{3}{4}x^2 + x + 4$.

(a) (i) Write down $f'(x)$.

(ii) Find the equation of the normal to the curve of f at $(2, 3)$.

(iii) This normal intersects the curve of f at $(2, 3)$ and at one other point P. Find the x-coordinate of P. *[9 marks]*

Part of the graph of f is given below.



(b) Let R be the region under the curve of f from $x = -1$ to $x = 2$.

(i) Write down an expression for the area of R.

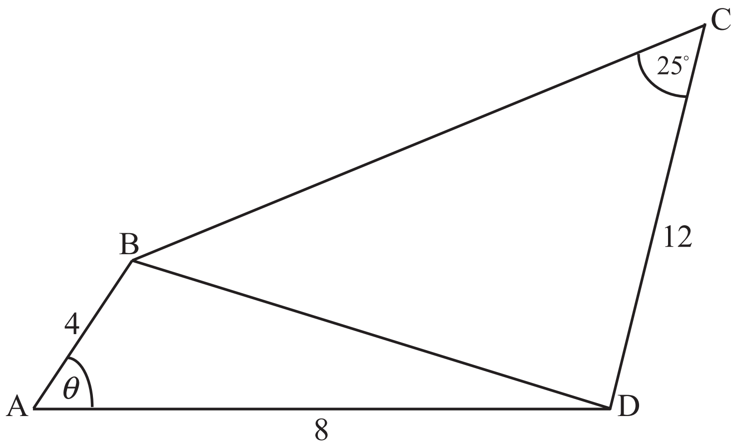
(ii) Calculate this area.

(iii) The region R is revolved through 360° about the x-axis. Write down an expression for the volume of the solid formed. *[6 marks]*

(c) Find $\int_1^k f(x)\,\mathrm{d}x$, giving your answer in terms of k. *[6 marks]*

2. *[Maximum mark: 16]*

The diagram below shows a quadrilateral ABCD. $\text{AB} = 4$, $\text{AD} = 8$, $\text{CD} = 12$, $\text{B}\hat{\text{C}}\text{D} = 25^\circ$, $\text{B}\hat{\text{A}}\text{D} = \theta$.



(a) Use the cosine rule to show that $\text{BD} = 4\sqrt{5 - 4\cos\theta}$. *[2 marks]*

Let $\theta = 40^\circ$.

(b) (i) Find the value of $\sin \text{C}\hat{\text{B}}\text{D}$.

(ii) Find the two possible values for the size of $\text{C}\hat{\text{B}}\text{D}$.

(iii) Given that $\text{C}\hat{\text{B}}\text{D}$ is an acute angle, find the perimeter of ABCD. *[12 marks]*

(c) Find the area of triangle ABD. *[2 marks]*

3. *[Total mark: 22]*

Part A *[Maximum mark: 8]*

Three students, Kim, Ching Li and Jonathan each have a pack of cards, from which they select a card at random. Each card has a 0, 3, 4, or 9 printed on it.

(a) Kim states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$\mathrm{P}(X = x)$	0.3	0.45	0.2	0.35

Explain why Kim is incorrect. *[2 marks]*

(b) Ching Li correctly states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$\mathrm{P}(X = x)$	0.4	k	$2k$	0.3

Find the value of k. *[2 marks]*

(c) Jonathan correctly states that the probability distribution for his pack of cards is given by $\mathrm{P}(X = x) = \frac{x+1}{20}$. One card is drawn at random from his pack.

(i) Calculate the probability that the number on the card drawn is 0.

(ii) Calculate the probability that the number on the card drawn is greater than 0. *[4 marks]*

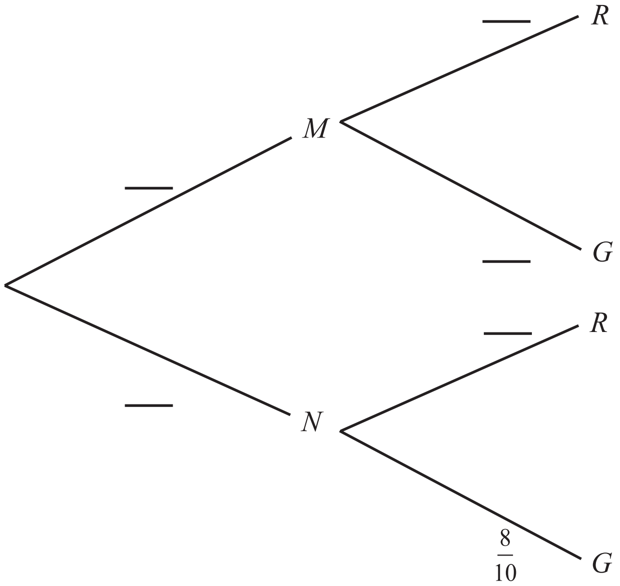
(This question continues on the following page)

(Question 3 continued)

Part B *[Maximum mark: 14]*

A game is played, where a die is tossed and a marble selected from a bag.
Bag M contains 3 red marbles (R) and 2 green marbles (G).
Bag N contains 2 red marbles and 8 green marbles.
A fair six-sided die is tossed. If a 3 or 5 appears on the die, bag M is selected (M).
If any other number appears, bag N is selected (N).
A single marble is then drawn at random from the selected bag.

(a) **Copy** and **complete** the probability tree diagram on **your answer sheet**.



[3 marks]

(b) (i) Write down the probability that bag M is selected and a green marble drawn from it.

(ii) Find the probability that a green marble is drawn from either bag.

(iii) Given that the marble is green, calculate the probability that it came from Bag M. *[7 marks]*

(c) A player wins \$ 2 for a red marble and \$ 5 for a green marble. What are his expected winnings? *[4 marks]*

4. *[Maximum mark: 12]*

(a) Consider the geometric sequence $-3, 6, -12, 24, \ldots$.

(i) Write down the common ratio.

(ii) Find the 15th term. *[3 marks]*

Consider the sequence $x-3$, $x+1$, $2x+8$, ….

(b) When $x=5$, the sequence is geometric.

(i) Write down the first three terms.

(ii) Find the common ratio. *[2 marks]*

(c) Find the other value of x for which the sequence is geometric. *[4 marks]*

(d) For this value of x, find

(i) the common ratio;

(ii) the sum of the infinite sequence. *[3 marks]*

5. *[Maximum mark: 19]*

The position vector of point A is $2\boldsymbol{i}+3\boldsymbol{j}+\boldsymbol{k}$ and the position vector of point B is $4\boldsymbol{i}-5\boldsymbol{j}+21\boldsymbol{k}$.

(a) (i) Show that $\overrightarrow{\mathrm{AB}}=2\boldsymbol{i}-8\boldsymbol{j}+20\boldsymbol{k}$.

(ii) Find the unit vector $\boldsymbol{u}$ in the direction of $\overrightarrow{\mathrm{AB}}$.

(iii) Show that $\boldsymbol{u}$ is perpendicular to $\overrightarrow{\mathrm{OA}}$. *[6 marks]*

Let S be the midpoint of [AB]. The line L_1 passes through S and is parallel to $\overrightarrow{\mathrm{OA}}$.

(b) (i) Find the position vector of S.

(ii) Write down the equation of L_1. *[4 marks]*

The line L_2 has equation $\boldsymbol{r}=(5\boldsymbol{i}+10\boldsymbol{j}+10\boldsymbol{k})+s(-2\boldsymbol{i}+5\boldsymbol{j}-3\boldsymbol{k})$.

(c) Explain why L_1 and L_2 are not parallel. *[2 marks]*

(d) The lines L_1 and L_2 intersect at the point P. Find the position vector of P. *[7 marks]*